The state in the copper samples on the pressure plate was determined by the reflection method with the use of the results of [4]: p=368 kbar, D=5.15 km/sec, u=0.80 km/sec, $\rho_0=8.93$ g/cm^3. A flying indicator was used for better observation of the motion of the surface. It was made of copper foil with a thickness of 0.03 mm and was attached to the sample surface by rubbing it on VM-4 oil. The measurements were carried out with an S1-24 oscilloscope. The supply source was a 10-μF capacitor, which was discharged into the sensing element circuit with a limiting resistance R = 400 Ω. The initial resistance of the elements in the test was 10-12 Ω.

The initial data and experimental results are summarized in Table 1. Also given in the table is the result of Al'tshuler et al. [1], which exhibits good agreement with our result.

It is evident from the given example that the proposed method can be used successfully in investigations of the properties of materials under high pressures. The application of the proposed method affords the possibility of determining unloading angles in complex shock waveforms and of determining wave profiles.

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ASYMPTOTIC OF THE FLOW IN THE NEIGHBORHOOD OF A CENTER DURING COLLAPSE OF A SPHERICAL CAVITY

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1. The problem of the collapse of a spherical cavity is examined in [1, 2] and the limit mode for gas flow outside the cavity in the neighborhood of the center is presented as the cavity radius tends to zero $R \to 0$.* The equation of state of the gas in the customary notation has the form

$$p = \rho_0 \frac{c_0^2}{\varkappa} (\delta^{\varkappa} S - 1), \tag{1.1}$$

where p is the pressure, ρ is the density, $\delta = \rho/\rho_0$, c is the speed of sound, S in an entropy quantity, and the subscript 0 corresponds to the unperturbed state. The flow up to the time of collapse was assumed isentropic $S=S_0$ in the approximation under consideration. Zero pressure, and therefore, constant, nonzero specific density $\delta = \delta_0$ and speed of sound $c_0/\sqrt{\delta_0}$ corresponded to the cavity boundary, where $\delta_0^{\varkappa} S_0 = 1$. Without limiting the generality, it can be considered that $S_0=\delta_0=1$ and, correspondingly, the sound speed on the free boundary equals c_0 because the gasdynamics equations in combination with the equation of state (1.1) are invariant relative to a similarity transformation:

$$r = r_0 r,\ t = t_0 t,\ u = u_0 u,\ \delta = \delta_0 \delta,\ S = S_0 S,$$

where

$$u_0 = r_0/t_0;\ S_0 \delta_0^{\varkappa} = 1;\ u_0 = 1/\sqrt{\delta_0}.$$

*All the personal references associated with the self-similar solution of the appropriate problem are presented in [1].

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It is well known from tests and numerical computations that high densities, and correspondingly pressures, occur in the neighborhood of the center as $R \to 0$. Hence the equation of state (1.1) was replaced by the equation of state of a polytropic gas

$$p = \frac{\rho_0 c_0^2 \delta^\varkappa S}{\varkappa} \qquad (1.2)$$

in obtaining the principal term of the flow asymptotic as $R \to 0$. It was assumed that the desired limit mode is independent of the initial data. In such a formulation, the problem was reduced to finding the self-similar solution of the gasdynamic equations corresponding to spherical symmetry and satisfying the boundary conditions on the free surface:

$$p = 0, \quad \frac{dR}{dt} = u, \qquad (1.3)$$

where u is the velocity and t the time measured from the instant of collapse.

Because of (1.2), the zero density, and therefore, the zero speed of sound, correspond to the zero pressure. Therefore the speed of sound on the free boundary in the self-similar solution found, which is the principal term of the asymptotic in the neighborhood of the center as $R \to 0$, was different by the finite constant c_0 in the initial formulation of the problem.

The second member of the asymptotic in the neighborhood of the center as $R \to 0$ is written down in this paper. The sum of the first two terms of the asymptotic turns out to be sufficient to obtain the given speed of sound c_0 in the asymptotic of the free boundary corrected by the second term as $R \to 0$. Hence, as before, the flow during collapse is assumed isentropic.

2. Because of the isentropicity of the flow in the equation of state (1.1) or (1.2), the gasdynamics equations corresponding to spherical symmetry can be represented as follows:

$$\frac{\partial A}{\partial t} + \alpha \frac{\partial A}{\partial r} = \frac{\beta^2 - \alpha^2}{2r}, \quad \frac{\partial B}{\partial t} + \beta \frac{\partial B}{\partial r} = \frac{\alpha^2 - \beta^2}{2r},$$

where A and B are Riemann invariants, and α and β are slopes of the characteristics

$$A = u + \frac{2}{\varkappa - 1} c, \quad B = u - \frac{2}{\varkappa - 1} c. \qquad (2.1)$$

We seek the asymptotic of the flow as $R \to 0$ in the form

$$A = \frac{r}{kt}\left[a_0(\xi) + r^\nu a_1(\xi)\right], \quad B = \frac{r}{kt}\left[b_0(\xi) + r^\nu b_1(\xi)\right], \qquad (2.2)$$

where

$$\xi = \xi_0 t r^{-k}. \qquad (2.3)$$

The line ξ =const=1 corresponds to the free surface in a self-similar flow, as can be seen by selecting the constant ξ_0 in an appropriate manner.

In the approximation proposed, the equation of the free boundary is represented in the form

$$\xi = 1 + gR^{\nu_1}. \qquad (2.4)$$

Because of (1.1), the conditions on the free boundary are rewritten from (1.3) in the form

$$\frac{\varkappa - 1}{4} \frac{R}{kt} \{a_0(\xi) - b_0(\xi) + R^\nu [a_1(\xi) - b_1(\xi)]\} \to c_0, \qquad (2.5)$$

$$\frac{R}{kt}\left(1 - \frac{\nu_1}{k} gR^{\nu_1}\right) \approx \frac{R}{2kt} \{a_0(\xi) + b_0(\xi) + R^\nu [a_1(\xi) + b_1(\xi)]\}$$

as $R \to 0$.

3. The functions $a_0(\xi)$ and $b_0(\xi)$ are the principal terms of the asymptotic corresponding to the self-similar flow and are determined by the system of ordinary differential equations

$$k\xi \frac{da_0}{d\xi} = \frac{2(h-1)ka_0 - (h+1)a_0^2 - (h-2)a_0 b_0 + b_0^2}{2(h-1) - ha_0 - (h-2)b_0}, \qquad (3.1)$$

$$k\xi \frac{db_0}{d\xi} = \frac{2(h-1)kb_0 - (h+1)b_0^2 - (h-2)a_0b_0 + a_0^2}{2(h-1) - hb_0 - (h-2)a_0} \quad \left(h = \frac{\varkappa+1}{\varkappa-1}\right). \tag{3.1}$$

The boundary conditions are imposed on the free surface to which the line $\xi = 1$ corresponds, and on the line $\xi = 0$ corresponding to a focusing slit.

As already noted, the sound of speed on the free boundary is zero in the self-similar solution. Hence, it follows from (1.3), (2.1) and (2.2) that for $\xi = 1$

$$a_0(1) = b_0(1) = 1, \tag{3.2}$$

while it follows from the condition of finiteness of the function on the focusing slit that for $\xi = 0$

$$a_0(0) = b_0(0) = 0. \tag{3.3}$$

As is known [1], the index k in the representation (2.4) cannot be determined from dimensional considerations but is found from the conditions for the existence of a solution of the system (3.1) which satisfies the boundary conditions (3.2) and (3.3). The desired integral curve should hence pass through the singularity of the system (3.1):

$$\xi = \xi_1 \quad (0 < \xi_1 < 1), \quad a_0(\xi_1) = \frac{2(h-1) - hb_0(\xi_1)}{h-2}, \tag{3.4}$$
$$b_0(\xi_1) = \frac{1}{4}\left[-(h-2)^2(k-1) + 2h - \sqrt{[(h-2)^2(k-1) - 2h]^2 - 16(h-1)}\right].$$

For values of the polytropic index $\varkappa$ belonging to the range $0 < \varkappa < 8.47$, the singularity at this point is of node type. Hence, for these values of $\varkappa$ it turns out that the possible values of k fill the whole band whose ends are determined by the polytropic index $\varkappa$. However, the solutions undergoing a weak discontinuity on the line $\xi = \xi_1$ corresponding to the characteristic arriving at the center at the time of collapse correspond to almost all of them. Because a weak discontinuity at the characteristic mentioned does not correspond to the physical formulation of the problem, "analytic" indices, i.e., those to which analytic solutions correspond, are taken as the desired values of the self-similarity index k. However it turns out that this requirement does not assure uniqueness of the solution; a discrete set of "analytic" indices exists [1].

The quantity k henceforth corresponds to some fixed "analytic" index of self-similarity for a given polytropic index $\varkappa$.

The asymptotic of the functions $a_0(\xi)$ and $b_0(\xi)$ as $\xi \to 1$ can be obtained from (3.1) and conditions (3.2):

$$a_0(\xi) \approx 1 - \sqrt{\frac{2(h-1)(k-1)(1-\xi)}{k}} + \frac{2}{h+1}\frac{(h-1)k - (h+2)}{k}(1-\xi),$$
$$b_0(\xi) \approx 1 + \sqrt{\frac{2(h-1)(k-1)(1-\xi)}{k}} + \frac{2}{h+1}\frac{(h-1)k - (h+2)}{k}(1-\xi) \tag{3.5}$$

and as $\xi \to 0$

$$a_0(\xi) \approx R_0\xi, \quad b_0(\xi) \approx Q_0\xi. \tag{3.6}$$

4. The functions $a_1(\xi)$ and $b_1(\xi)$ are determined by the equations

$$a_1'\xi\left(1 - \frac{\varkappa+1}{4}a_0 - \frac{3-\varkappa}{4}b_0\right) - a_1\left[1 - \frac{\varkappa+1}{4}\left(\frac{1}{k}a_0 - a_0'\xi\right)\right.$$
$$\left. - \frac{1+\nu}{k}\left(\frac{\varkappa+1}{4}a_0 + \frac{3-\varkappa}{4}b_0\right) - \frac{\varkappa-1}{2k}a_0\right] + b_1\left[\frac{3-\varkappa}{4}\left(\frac{a_0}{k} - a_0'\xi\right) - \frac{\varkappa-1}{2k}b_0\right] = 0,$$
$$b_1'\xi\left(1 - \frac{\varkappa+1}{4}b_0 - \frac{3-\varkappa}{4}a_0\right) - b_1\left[1 - \frac{\varkappa+1}{4}\left(\frac{1}{k}b_0 - b_0'\xi\right)\right. \tag{4.1}$$
$$\left. - \frac{1+\nu}{k}\left(\frac{\varkappa+1}{4}b_0 + \frac{3-\varkappa}{4}a_0\right) - \frac{\varkappa-1}{2k}b_0\right] + a_1\left[\frac{3-\varkappa}{4}\left(\frac{b_0}{k} - b_0'\xi\right) - \frac{\varkappa-1}{2k}a_0\right] = 0.$$

The values $\xi = 1$, $\xi = \xi_1$ and $\xi = 0$ will be singular points for this system. To obtain the asymptotic of the functions $a_1(\xi)$ and $b_1(\xi)$ in the neighborhood of these values the coefficients for these functions and their derivatives in system (4.1) are replaced by their expansions in the neighborhood of these values.

If the appropriate coefficients of system (4.1) are replaced by the first terms of their expansions in the neighborhood of the value $\xi = 1$ by using the asymptotic (3.5), then the corresponding equations will have the form

$$\frac{\varkappa-1}{2}(1-\xi)a_1' = \frac{\varkappa+1}{8}a_1 + \frac{3-\varkappa}{8}b_1, \quad \frac{\varkappa-1}{2}(1-\xi)b_1' = \frac{3-\varkappa}{8}a_1 + \frac{\varkappa+1}{8}b_1, \tag{4.2}$$

from which

$$\frac{db_1}{da_1} = \frac{(\varkappa+1)b_1 + (3-\varkappa)a_1}{(3-\varkappa)b_1 + (\varkappa+1)a_1}.$$

The general solution of this equation will be

$$b_1 - a_1 = C(b_1 + a_1)^{(\varkappa-1)/2}. \tag{4.3}$$

The system (4.2) does not possess a bounded solution for $\xi = 1$, except the trivial $a_1 \equiv b_1 \equiv 0$. Indeed, it is necessary that $a(1) = b(1) = 0$ for the solution to be bounded at $\xi = 1$. Substituting $b_1 \approx a_1$ or $b_1 \approx -a_1$ in (4.2) according to the solution (4.3), we obtain in the first case

$$a_1 \approx C(1-\xi)^{-1/(\varkappa-1)}, \; b_1 \approx C(1-\xi)^{-1/(\varkappa-1)},$$

and in the second

$$a_1 \approx D(1-\xi)^{-1/2}, \; b_1 \approx -D(1-\xi)^{-1/2}.$$

For $\varkappa = 3$ the following asymptotics are possible:

$$a_1 \approx C_1(1-\xi)^{-1/2}, \; b_1 \approx C_2(1-\xi)^{-1/2},$$

where, in general, $C_1 \neq C_2$. Because of taking account of the next terms in the expansions of the coefficients of system (4.1), the asymptotic of the general solution of this system will appear as follows in the neighborhood of the value $\xi = 1$:

$$\begin{gathered} a_1 \approx M(1-\xi)^{-1/(\varkappa-1)} + K(1-\xi)^{(\varkappa-3)/2(\varkappa-1)} + \ldots + N(1-\xi)^{-1/2}, \\ b_1 \approx M(1-\xi)^{-1/(\varkappa-1)} - K(1-\xi)^{(\varkappa-3)/2(\varkappa-1)} + \ldots - N(1-\xi)^{-1/2} \text{ for } \varkappa \neq 3, \\ a_1 \approx M_1(1-\xi)^{-1/2}, \; b_1 \approx M_2(1-\xi)^{-1/2} \text{ for } \varkappa = 3, \end{gathered} \tag{4.4}$$

where $K = \frac{2M}{2-\varkappa}\left[\frac{2-\varkappa}{\varkappa}\frac{(h-1)k-(h+2)}{k} + 1 - \frac{\nu+2}{k}\right]$; M, N, M_1, M_2 are arbitrary constants. For $\xi = \xi_1$ the coefficient of $b_1'\xi$ vanishes. Since the functions $a_0(\xi)$ and $b_0(\xi)$ are analytic in the neighborhood of the value $\xi = \xi_1$, then the equations

$$a_1' \approx a_1 B + b_1 D, \qquad b_1' \approx \frac{a_1 A + b_1 C}{\xi - \xi_1}$$

are obtained as a result of replacing the coefficients of system (4.1) by the first terms of their expansions in the neighborhood of ξ_1, where A, B, C, D are constants determined by the self-similar solution. The solution of system (4.1), which is bounded in the neighborhood of the line $\xi = \xi_1$, exists under the condition

$$a_1(\xi_1)A + b_1(\xi_1)C = 0. \tag{4.5}$$

Two analytic integral curves of system (4.1) issue from the point ξ_1, $a_1(\xi_1)$, $b_1(\xi_1)$. One corresponds to the solution $\xi = \xi_1$ and lies in the plane $a_1 b_1$, and the second corresponds to the curve L which tends to infinity as $\xi \to 1$ in conformity with the asymptotic (4.4). Because of the linearity and homogeneity of system (4.1), the two integral curves corresponding to different solutions of (4.5) differ by a constant factor.

According to (3.3) for $\xi = 0$ the functions $a_0(\xi)$, $b_0(\xi)$ vanish. Hence, it follows from system (4.1) that the functions $a_1(\xi)$ and $b_1(\xi)$ also vanish for $\xi = 0$ according to the asymptotic

$$a_1(\xi) \approx R_1\xi, \; b_1(\xi) \approx Q_1\xi, \tag{4.6}$$

where R_1 and Q_1 are constants.

5. According to the asymptotics (3.4) and (4.4) obtained, the conditions (2.5) on the free boundary appear as follows:

$$\frac{\varkappa-1}{4k}R^{1-k}\xi_0\left\{-2\sqrt{\frac{2(h-1)(k-1)(1-\xi)}{k}} + R^{\nu}O\left[(1-\xi)^{\min\left(-\frac{1}{2},\,(\varkappa-3)/2(\varkappa-1)\right)}\right]\right\} \to c_0,$$

$$\frac{R}{kt}\left(1 - \frac{\nu_1}{k}gR^{\nu_1}\right) \approx \frac{R}{kt}\left\{1 + \frac{2}{h+1}\frac{(h-1)k-(h+2)}{k}(1-\xi) + M(1-\xi)^{-1/(\varkappa-1)}R^{\nu}\right\} \text{ for } R \to 0$$

(2M and 3N are replaced by M_1+M_2 and M_1-M_2 for $\varkappa$). Taking (2.4), the equation of the free boundary into account, these conditions will be satisfied if and only if

$$\nu = \frac{\varkappa}{\varkappa-1}\nu_1.$$

Since $\nu + \min\left(-\frac{\nu_1}{2}, \frac{(\varkappa-3)\nu_1}{2(\varkappa-1)}\right) > \frac{\nu_1}{2}$, here, then

$$-\frac{(\varkappa-1)\xi_0}{2\varkappa}\sqrt{\frac{-2(h-1)(k-1)}{k}}\,g = c_0, \quad \nu_1 = 2(k-1); \tag{5.1}$$

$$\left\{\frac{\nu_1}{k} + \frac{2}{(h+1)k}[(h-1)k-(h+2)]\right\}g + M(-g)^{-1/(\varkappa-1)} = 0. \tag{5.2}$$

The constants g_0 and M_0 satisfying (5.1) and (5.2) determine the desired solutions $a_1(\xi)$ and $b_1(\xi)$. Indeed, as $\xi \to 1$ the asymptotic of the integral curve of system (4.1) emerging from the point ξ_1, $a_1(\xi_1)$, $b_1(\xi)$, where $a_1(\xi_1)$ and $b_1(\xi_1)$ are any values satisfying condition (4.5), is determined by (4.4). The desired solution is obtained as a result of multiplying the values $a_1(\xi_1)$ and $b_1(\xi)$ by a constant factor M_0/M_1 for $\varkappa \neq 3$ and $2M_0/M_1 + M_2$ for $\varkappa = 3$, as is possible because of the homogeneity and linearity of system (4.1). The solution obtained can be continued into the interval $\xi_1 > \xi > 0$, where the functions $\xi \to 0$ and $a_1(\xi)$ vanish as $b_1(\xi)$ according to the asymptotic (4.6).

6. In constructing the self-similar solution corresponding to the flow after the time of collapse, it should be kept in mind that the self-similar solution cannot be continued continuously into the whole neighborhood of the center for $t > 0$. A reflected shock issuing from the center occurs to which the $\xi = \xi_B = \text{const}$ corresponds because of self-similarity. Hence, the shock velocity D is determined by the formula

$$D = r/kt.$$

The velocity and the speed of sound in the self-similar flow are represented according to (2.2) in the form

$$u = \frac{r}{kt}\frac{a_0(\xi)+b_0(\xi)}{2} = \frac{r}{kt}U_0(\xi), \quad c = \frac{r}{kt}\frac{a_0(\xi)-b_0(\xi)}{2(h-1)} = \frac{r}{kt}C_0(\xi).$$

From the condition on the shock front

$$\frac{\rho_{01}}{\rho_{00}} = \frac{D-U_{00}}{D-U_{01}} = \frac{1-U_{00}(\xi_B)}{1-U_{01}(\xi_B)}$$

(the second 0 subscript corresponds to the value of the function ahead of the front, and the 1 to behind the front) it follows that the ratio of the densities is constant on the front. Since the speed of sound is not zero ahead of the front, then the ratio of the speeds of sound will also be constant. The ratio of the entropy quantities will also be constant because of the equation of state (1.2). The entropy ahead of the front was constant, it will therefore be constant in the self-similar flow and behind the shock.

Equations for the functions $U_0(\xi)$ and $C_0(\xi)$ follow from system (3.1)

$$\frac{dU_0}{dC_0} = (h-1)\frac{C_0[(h-1)C_0(k-U_0)-3C_0U_0]-(1-U_0)\left[U_0^2+(h-1)C_0^2-kU_0\right]}{C_0\{(1-U_0)[(h-1)(k-U_0)-3U_0]-\left[U_0(U_0-k)+(h-1)C_0^2\right]\}},$$

$$\frac{d\xi}{dC_0} = \frac{k\xi}{C_0}\frac{[C_0 dU_0/dC_0-(h-1)(1-U_0)]}{[3U_0-(h-1)(k-U_0)]}, \tag{6.1}$$

and conditions on the shock front have the form

$$\frac{1-U_{00}(\xi_B)}{1-U_{01}(\xi_B)} = \frac{C_{00}^2(\xi_B)+\varkappa[1-U_{00}(\xi_B)]^2}{C_{01}^2(\xi_B)+\varkappa[1-U_{01}(\xi_B)]^2} = \frac{(\varkappa+1)[1-U_{00}(\xi_B)]C_{01}^2(\xi_B)+(\varkappa-1)[1-U_{01}(\xi_B)]C_{00}^2(\xi_B)}{(\varkappa-1)[1-U_{00}(\xi_B)]C_{01}^2(\xi_B)+(\varkappa+1)[1-U_{01}(\xi_B)]C_{00}^2(\xi_B)}. \tag{6.2}$$

The line $\xi = -\infty$ corresponds to the center $r = 0, t > 0$. Since the velocity is zero at the center, and the speed of sound is finite, then $C_0(-\infty) > U_0(-\infty)$. Taking this into account, the asymptotic of the functions can be obtained from (6.1) as $\xi \to -\infty$

$$U_0 \approx A_0 + A_1(-\xi)^{-2/k}, \; C_0 \approx F_0(-\xi)^{1/k} + F_1(-\xi)^{-1/k}, \tag{6.3}$$

where $A_0 = \frac{1}{3}(h-1)(k-1)$; $A_1 = \frac{A_0(1-A_0)(k-A_0)}{5F_0^2}$; $F_1 = -\frac{A_0(A_0-k)}{2(h-1)F_0}$. The constant F_0 is determined from merging the solutions ahead of the shock front by conditions (6.2). The shock front is found as follows. The integral curves of the first equation of system (6.1) are determined: L_1 leaving the point $U_0 = C_0 = 0$ towards $C_0 > 0$ in a direction determined by the asymptotic (3.6):

$$U_0 = C_0 = 0, \quad \frac{dU_0}{dC_0} = (h-1)\frac{R_0 + Q_0}{R_0 - Q_0},$$

and L_2 leaving the point $U_0 = (1/3)(h-1)(k-1)$, $C_0 = \infty$. The curve L_3 ($U_{01} = f(C_{01})$) is constructed according to (6.2) from values of the quantities on the integral curve L_1. The point of intersection of the curves L_2 and L_3 corresponds to values of the functions $U_{01}(\xi_B)$ and $C_{01}(\xi_B)$ on the shock front. The quadrature realized according to the second equation of system (6.1) determines the value of the quantity ξ_B and the constant F_0 in the asymptotic (6.3). The value of the entropy S behind the shock front is determined by the formula

$$S = S_{01} = \left(\frac{1-U_{01}}{1-U_{00}}\right)^{\varkappa-1} \frac{C_{01}^2}{C_{00}^2} S_{00}.$$

7. The second term of the asymptotic as $t > 0$ can be obtained as a continuous continuation of the solution found in Sec. 5 to just the value $\xi = \xi_B$ corresponding to the reflected shock front in the self-similar solution. Taking account of the second term of the asymptotic, the line

$$\xi = \xi_B + dr^{\nu}$$

will correspond to the reflected shock front. The flow behind the shock front will already not be isentropic in the approximation under consideration. The desired functions (there are now three, the velocity u, the speed of sound c, and the entropy S) are represented in the form

$$u = \frac{r}{kt}[U_0(\xi) + r^{\nu}U_1(\xi)], \quad c = \frac{r}{kt}[C_0(\xi) + r^{\nu}C_1(\xi)], \qquad (7.1)$$
$$S = S_0[1 + r^{\nu}S_1(\xi)/S_0].$$

Before the front $S_0 = S_{00} = 1$ is constant; behind the front

$$S_0 = S_{01} = \left[\frac{1-U_{00}(\xi_B)}{1-U_{01}(\xi_B)}\right]^{1-\varkappa} \frac{C_{01}^2(\xi_B)}{C_{00}^2(\xi_B)} S_{00}.$$

The equations for the functions $U_1(\xi)$, $C_1(\xi)$ and $S_1(\xi)$ have the form

$$(1-U_0)U_1'\xi - (h-1)C_0C_1'\xi + \frac{h-1}{2\varkappa}\frac{C_0^2}{U_0}S_1'\xi + \frac{2+\nu}{k}U_0U_1 + (h-1)\left(\frac{2+\nu}{k}C_0 - C_0'\xi\right)C_1 - (U_0'\xi + 1)U_1 = 0,$$

$$C_0U_1'\xi - (h-1)(1-U_0)C_1'\xi - \left[\frac{h+2+\nu(h-1)}{k}U_0 - (h-1) - U_0'\xi\right]C_1 - \left[\frac{h+2+\nu}{k}C_0 - (h-1)C_0'\xi\right]U_1 = 0, \qquad (7.2)$$

$$S_1'\xi + \frac{\nu U_0}{(1-U_0)k}S_1 = 0.$$

The solutions of (7.2) should satisfy the requirement

$$\frac{r^{\nu}U_1(\xi)}{U_0(\xi)} \to 0, \quad \frac{r^{\nu}C_1(\xi)}{C_0(\xi)} \to 0, \quad \frac{r^{\nu}S_1(\xi)}{S_0(\xi)} \to 0 \quad \text{for} \quad r \to 0,$$

$t \to 0$ and $-\infty < \xi < \xi_B$. Two solutions satisfying this requirement exist:

$$\{U_{1a}(\xi), C_{1a}(\xi), S_{1a}(\xi)\}, \{U_{1B}(\xi), C_{1B}(\xi), S_{1B}(\xi)\}.$$

To the accuracy of a constant, the first is extracted by the asymptotic for $\xi \to -\infty$:

$$U_{1a}(\xi) \approx B_1(-\xi)^{\beta-2/k}, \quad C_{1a} \approx B_2(-\xi)^{\beta+1/k} + D_2(-\xi)^{\beta-1/k}, \qquad (7.3)$$
$$S_{1a} \approx B_3(-\xi)^{\beta} + D_3(-\xi)^{\beta-2/k},$$

where $\beta = -\frac{\nu(h-1)(k-1)}{3-(h-1)(k-1)}$, and the constants B_i, D_i are connected by the following relationships:

$$B_2 = \frac{F_0}{2\varkappa} B_3,$$
$$D_2 = \frac{k\beta A_0(k-A_0)(k\beta-2) - 10k\beta F_0(2A_0F_1 - A_1F_0) - 10(2+\nu-k\beta)A_0^2F_0F_1}{10(2+\nu-k\beta)A_0^2F_0^2} B_2,$$
$$D_3 = \frac{k\beta(k-A_0)}{10F_0^2} B_3, \tag{7.4}$$
$$B_1 = \frac{[(h-1)(\nu-k\beta)+5]A_1B_2 - 2(h-1)(1-A_0)D_2}{(k\beta-\nu-5)F_0}.$$

In the second solution the function is $S_{1B}(\xi) \equiv 0$, and the functions $U_{1B}(\xi)$ and $C_{1B}(\xi)$ tend to infinity as $\xi \to -\infty$ in conformity with the asymptotics:

$$U_{1B}(\xi) \approx L_1(-\xi)^{\nu/k}, \quad C_{1B} \approx L_2(-\xi)^{(1+\nu)/k}. \tag{7.5}$$

The coefficients L_1 and L_2 are connected by the relationship

$$L_1 = -\frac{(h-1)\nu}{3F_0} L_2. \tag{7.6}$$

The asymptotics, and therefore, the solutions themselves because of the linearity, are determined to the accuracy of a constant. For definiteness, we set $B_3=1$ in (7.3) and (7.4), and $L_2=1$ in (7.5) and (7.6). The desired solution is a linear combination of the solutions of the system (7.2) which have the asymptotics (7.3) and (7.5) as $\xi \to -\infty$:

$$\{U_1(\xi), C_1(\xi), S_1(\xi)\} = q_1\{U_{1a}(\xi), C_{1a}(\xi), S_{1a}(\xi)\} + q_2\{U_{1B}(\xi), C_{1B}(\xi), S_{1B}(\xi)\}.$$

The constants q_1, q_1, and d are determined by conditions on the shock front. Since the shock velocity in the approximation under consideration has the form

$$D = \frac{r}{kt}\left[1 - \frac{\nu d}{k\xi_B} r^\nu\right],$$

then the relationships between the second terms on the shock front are represented by three equations

$$\frac{\left(\frac{\nu}{k\xi_B}+U'_{01}\right)d+U_{11}}{1-U_{01}} - \frac{\left(\frac{\nu}{k\xi_B}+U'_{00}\right)d+U_{10}}{1-U_{00}} = \frac{(\varkappa+1)\alpha+(\varkappa-1)\beta}{(\varkappa+1)C_{01}^2(1-U_{00})+(\varkappa-1)C_{00}^2(1-U_{01})}$$

$$-\frac{(\varkappa-1)\alpha+(\varkappa+1)\beta}{(\varkappa-1)C_{01}^2(1-U_{00})+(\varkappa+1)C_{00}^2(1-U_{01})} = \frac{2C_{00}(C'_{00}d+C_{10})+\varkappa\{(1-U_{00})[(U'_{01}-U'_{00})d+U_{11}-U_{10}]}{C_{00}^2+\varkappa(1-U_{00})(U_{01}-U_{00})}$$

$$-\frac{(U_{01}-U_{00})\left[\left(\frac{\nu}{k\xi_B}+U'_{00}\right)d+U_{10}\right]\}}{C_{00}^2+\varkappa(1-U_{00})(U_{01}-U_{00})} - 2\left(\frac{C'_{01}d+C_{11}}{C_{01}}\right) = \frac{1}{\varkappa-1}\left[2\left(\frac{C'_{01}d+C_{11}}{C_{01}} - \frac{C'_{00}d+C_{10}}{C_{00}}\right) - \frac{S_{11}}{S_{01}}\right],$$

$$\alpha = 2(C'_{01}d+C_{11})(1-U_{00}) - C_{01}^2\left[\left(\frac{\nu}{k\xi_B}+U'_{00}\right)d+U_{10}\right],$$

$$\beta = 2(C'_{00}d+C_{10})(1-U_{01}) - C_{00}^2\left[\left(\frac{\nu}{k\xi_B}+U'_{01}\right)d+U_{11}\right],$$

where U_{00}, C_{00}, S_{00}, U_{10}, C_{10}, S_{10} are values of the self-similar solution and the second approximation ahead of the shock front and U_{01}, C_{01}, S_{01}, U_{11}, C_{11}, S_{11} are, correspondingly, behind the shock front for $\xi = \xi_B$ and the prime denotes the derivative with respect to ξ at $\xi = \xi_B$. As a result of the substitution

$$U_{11} = q_1U_{1a}(\xi_B) + q_2U_{1B}(\xi_B), \quad C_{11} = q_1C_{1a}(\xi_B) + q_2C_{1B}(\xi_B),$$
$$S_{11} = q_1S_{1a}(\xi_B) + q_2S_{1B}(\xi_B);$$

a system of linear inhomogeneous equations with three unknowns q_1, q_2, and d is obtained. The coefficients of these equations are smooth functions of the index $\varkappa$ and the self-similarity index k and can be obtained only by numerical integration, hence it should be expected that the determinant of the system is different from zero in the general case, as is confirmed by a numerical counting of the individual variants. If the determinant of the system should happen to equal zero, then perhaps it would be for exceptional values of $\varkappa$ and k. The set-up of the shock front and finding the gasdynamic functions behind it in a second approximation is terminated completely by the determination of q_1, q_2, and d.

It follows from the asymptotic formulas (7.3) and (7.5) and the representation of the functions that the values of the additions for the velocity and the entropy at the center ($r=0$, $t \geq 0$) are zero.

Lutskii performed a numerical computation of the second term of the asymptotic for values of the index $\varkappa=3$ ($k=1.411332$, $\nu=1.233996$, $\nu_1=0.822664$). Values of the constant coefficients are obtained as a result of numerical integration for the asymptotic formulas (2.4), (2.2) and (7.1) governing the shape of the free boundary, the shock front, and the values of the gasdynamic functions as $r \to 0$, $t \to 0$: The equation of the free boundary is

$$t \approx \frac{1}{\xi_0} r^k \left[1 - 3.417 \frac{c_0^2}{\xi_0^2} r^{\nu_1}\right],$$

the values of the gasdynamic functions on the slit are

$$u \approx \xi_0 r^{1-k} \left[0.5150 + 9.463 \frac{c_0^3}{\xi_0^3} r^{\nu}\right],$$

$$c \approx -\xi_0 r^{1-k} \left[0.5480 + 8.514 \frac{c_0^3}{\xi_0^3} r^{\nu}\right],$$

the equation of the shock front is

$$t \approx \frac{1}{\xi_0} r^k \left[-0.6132 + 6.8502 \frac{c_0^3}{\xi_0^3} r^{\nu}\right],$$

and the values of the gasdynamic functions on the shock front are

$$u \approx -\xi_0 r^{1-k} \left[0.1926 - 7.219 \frac{c_0^3}{\xi_0^3} r^{\nu}\right],$$

$$c \approx -\xi_0 r^{1-k} \left[1.371 + 40.01 \frac{c_0^3}{\xi_0^3} r^{\nu}\right], \quad S \approx 1.2049 + 5.1547 \frac{c_0^3}{\xi_0^3} r^{\nu}.$$

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